

So this lecture is on image quality assessment.  
I would say this lecture is a landmark.  
After this one, we will move into imaging modalities.  
Then your green textbook is the foundation.  
But I will use my slides and give you  
some state of the art development.  
Image quality assessment is the last piece of the foundation.  
And we're dealing with imaging modalities.  
So you produce images.  
So you want to know if the images are good or bad.  
So we will discuss different aspects of image quality  
assessment.  
And also, yesterday I uploaded chapter 7 on image quality.  
So that really finished the book draft of medical imaging.  
The part one and mathematical foundation.  
So I think the draft is draft.  
We need to spend time to make it better.  
One student is working with me trying to set up a teaching  
website.  
And this takes time.  
So any of you interested in some more time  
want to work with me helping, say,  
policy and homework solution and the figure editing website  
development.  
And there are some possibilities so we can discuss.  
And this is paid half time or full time if you're interested.  
But I really need a very good editorial or drawing scale  
to make things neat.  
I imagine after a few iterations,  
this will be a really good textbook.  
And the foundation part, as you see,  
I'm trying to cover the fundamental principle.  
And you know why you have Fourier transform and sampling  
theorem and so on.  
So I try to explain to the level feasible for BME  
undergraduate students.  
So this is a long term effort, but I really enjoy teaching.  
And particularly, I want to make the book  
as good as possible.  
And right now, it's very rough, I know.  
And some students read it.  
They say they can follow, but they need more examples.  
So I haven't got time to prepare homework solutions  
for the book yet.  
But anyway, this is just an opportunity.  
Maybe one of you could be good fit.  
Just let me know.  
We are on track and as we scheduled early on.  
So next lecture will be on x-ray physics and radiography.  
So take one image.  
And you'll have your first book chapter,  
I think the first nine sections.  
And everything before, I think section 10,  
computed tomography.  
So you can review.  
And then the next lecture, I wouldn't be available.  
I got an invitation to present some polarized nuclear imaging  
stuff.  
And we will know nuclear physics and nuclear imaging.  
I have some ideas about so-called polarized radio  
tracer-based imaging.  
And Siemens is interested, so I got an invitation.  
So I will fly to Idaho and I will be back very late Sunday.  
So I wouldn't be able to tease this lecture.

However, I think it doesn't matter.  
And we can use the video recording last time I teased this.  
So same, maybe I can just update slightly.  
Basically, same PowerPoint slides.  
And you watch the video and just imagine  
I'm just here, just virtual me a year ago.  
And I told actually physics.  
And actually physics doesn't change, so still same thing.  
So that part, you still attend the lecture,  
just to let you know it will change.  
And I will pick up, teach CT reconstruction, CT scanner.  
So everything from beginning until today  
is a foundational part.  
I should make this blue as well.  
So I'm pretty happy with the logic.  
I just need time to polish the textbook.  
And image quality and the diagnostic performance  
is a central topic I will tease in my way.  
And the textbook way is shown here.  
It's chapter five.  
I moved it early on.  
And once you know image quality assessment, any modernity,  
more or less the general concept of resolution, artifacts,  
signal to noise, result, all these concepts hold valid.  
So you can read the chapters.  
This is the easy reading.  
And I have many slides for this lecture.  
But the basic concepts are not hard.  
So if you listen carefully, you will know what's going on.  
So what I'm going to tease today, mainly three aspects.  
As outlined here, first I talk about some general measurement.  
You have images.  
You have ground truth.  
You want the images.  
You reconstruct it.  
You take as close as possible to the ground truth.  
Then you measure difference.  
The difference, and we use the concept of vector space  
to point there's some distance.  
And distance, most famous one, Euclidean distance.  
You have other distances.  
And you can measure how different they are  
or how similar two images are.  
So I have a distance measurement mainly  
mean squared error.  
That's Euclidean distance.  
I mentioned briefly so-called KL distance.  
That's a little tricky.  
So I put three green dots.  
Just to let you know, I will not test.  
Just to let you know.  
Then a very practical concept called similarity,  
structural similarity, as I am in the water idea.  
This paper is amazing.  
And the paper got over 10,000 citations, widely used.  
So if you ever write such a paper,  
you should be very proud of yourself.  
But we can't explain why this is so popular,  
what's the idea behind it as I am.  
So this is a general measurement.  
Then we move to system specific.  
We're talking about a system like a camera.  
It's an X-ray camera, a nuclear imager, ultrasound scanner.  
All these are kind of a camera.

You take a picture inside a human body.  
Then we have system specifications  
and how noisy the images would be.  
So you can detect a signal from a noisy background.  
And what's the resolution?  
The resolution has at least four aspects.  
Spatial resolution, small dots.  
You can tell them apart, but I will explain.  
Artifacts are structures in the images.  
However, these are not real structures.  
They're kind of like ghosts.  
You see things there, but they're really not there.  
You need to understand why you have artifacts, how  
many types of artifacts.  
So this lecture gives you a general concept.  
And when we learn particular medical imaging modality,  
you will see modality-related artifacts.  
So system specification, more system-oriented.  
Just to tell you how good my camera is.  
And this iPhone X camera, you have very high resolution.  
And the last part is not so much about how good your camera.  
And this question is a little bit different.  
It's called task-based assessment,  
the task-specific.  
So when I use the imager to handle  
a particular clinical task.  
Just see if your bone is broken, if you have any tumor in lung.  
So those are clinical tasks.  
The camera may be good or bad, but after all,  
if you can always tell your bone is not broken, don't worry.  
Just lay down, have a good rest.  
If you finish the task, the clinical task, successfully,  
then we consider the whole imaging workflow is perfect.  
So this is something ultimately we, as a patient or doctor,  
are concerned, task-based assessment.  
These three aspects are related, but they are not the same.  
So I will explain in detail.  
But first, I'll talk about general measurement,  
so the part of what.  
So mean squared, you know this.  
You have two images,  $y$ ,  $y$ -tailed.  
You have the different pixels indexed by  $i$ .  
You have  $n$ , small  $n$ , maybe 512 by 512.  
You just compare pixel-wise, and then you just sum all the arrows,  
and then you do average.  
This is easy.  
A little math here.  
So you see, you have this parameter,  
and then you have a truth.  
You estimate what's the true parameter.  
The estimation is  $\hat{\theta}$ .  
And then you can do so many, many times.  
So mean, mean squared arrow.  
So arrow is squared.  
This is the arrow, the difference between estimate  
and the truth, squared.  
That is the mean.  
 $E$  means expectation.  
This is the kind of mean.  
So you do a little bit of algebraic operation.  
Just do a little bit of operation here.  
You minus the mean of your estimation.  
You plus the same thing.  
Then you do expansion.

So you have this squared, you got this.  
It just goes through the algebraic steps.  
You'll find out that the mean squared arrow has two terms.  
One is variation.  
So that is the mean here.  
And fluctuation around mean for a particular estimate.  
So you just do the average.  
This is called a variance.  
So you know.  
The other thing is called a bias.  
Bias is the difference between true parameter  
and the estimated parameter of the average.  
So this is a bias.  
This is just two concepts.  
So you can go through yourself, just the mathematical steps.  
Something I highlighted here.  
And then you can try to understand yourself.  
So this is easy to appreciate.  
And there are multiple variants.  
So you have this mean squared arrow just explained.  
Or you say root mean squared arrow.  
Make the unit the same.  
So you got the square root operation.  
And if you say the squared root or the squared arrow,  
emphasize larger difference quadratically.  
So the difference is 100.  
Then after squared, it becomes 10,000.  
So you really penalize bigger amplitude very seriously.  
So you don't want to do that.  
You can do mean absolute arrow.  
And also called  $L_1$ .  
It's called  $L_2$  norm,  $L_1$  norm.  
It just uses absolute value.  
Or you just compute the arrow percentage-wise.  
So there are different things.  
And this measure, the distance measure, as you follow me.  
And it's very reasonable.  
Basically, you have, say, one image or one signal.  
And you have a standard.  
Just compare pixel-wise.  
So the measure is really relevant to the area  
between two curves, or between two surfaces,  
or between two volumes.  
It's the same thing.  
So we can measure.  
This is very, very reasonable.  
Is that all?  
So we really need more.  
And you will see this is just step one.  
Just try to think of how you measure two things  
and say they are so similar, or they are so much different,  
how you measure.  
This is one way.  
And when you have a probability function,  
this is not just for your information.  
So don't be bothered if you don't understand.  
You see this kind of strange combination.  
And you have two probability functions.  
The  $P$  is  $Q$ . And say the question is,  
how do you measure the difference between two  
probability distributions?  
Certainly, you can say, let's measure using Euclidean distance.  
That's one way.  
But a better way called call back Leibler distance,

KIO distance, is much more meaningful related to information processing.  
And this is beyond the scope of this lecture, just to let you know.  
There are such things called information divergence. And that's another kind of distance.  
And for P and Q, the distance, the KIO distance, always not negative.  
And they become zero.  
That means the distance becomes zero.  
And only if P and Q are identical.  
And this distance is not symmetric, not like, say, you go from Albany to New York City.  
Or you go back.  
The distance is the same, right?  
But for information distance, from P to Q is different from the distance from Q to P.  
And this is not that surprising.  
Like you climb mountains.  
You go way up, it's harder than the way down.  
This is some asymmetric property.  
So just to let you know for your knowledge.  
And this is related to so-called mutual information.  
So mutual information between two random variables,  $x$ ,  $y$ , is defined as KIO distance between the joint distribution and the marginal distribution multiplied together.  
So another way to view it is something like, say, the two variables.  
That's the idea.  
And then you do some measurement about one variable.  
So you get some information.  
Once you know certain thing about  $x$ , then how much you know about  $y$ ?  
If they are totally independent, and you say you measure  $x$ , and then nothing you know about  $y$ .  
So this is totally independent.  
If you know  $x$ , you completely know  $y$ .  
That's just a dependent thing.  
So it's just a deterministic relationship.  
So the more general thing is really something between.  
You have variable  $x$ , variable  $y$ .  
Statistically, how they correlate together.  
So normally, you use a linear correlation.  
But in terms of information theory, you use mutual information.  
So mutual information is defined in terms of entropy.  
And entropy is another very fancy concept, the fundamental concept behind information theory.  
So you have a probability distribution.  
The distribution is very uniform.  
Then you have a high level of uncertainty about the variable.  
On the other hand, the distribution is a delta function.  
So you know the outcome must be the case.  
So there are no ambiguity.  
So in the first case, entropy will be big.  
The level of uncertainty will be big.  
In the other case, entropy is low.  
So something about information processing.  
So the distance, the KL distance, is also very useful.  
Just to let you know that, but do not worry too much.  
Three green buttons.  
As a return to the least square measurement, we say least square measurement, like a mean squared error,

and a very reasonable, but it's not good enough.  
Why not good enough?  
Let me give you an example.  
So right above the picture is a true thing.  
Like you use all good imaging conditions.  
You got this.  
And you use different cameras under different conditions.  
The images can be noisy, and it can be very blurry,  
or whatever.  
You got five different versions.  
And they are not the same as the original one.  
And we want to have a measure how these images differ,  
or how they are similar.  
And certainly, you say, how about you just explain  
the mean squared error.  
We use that one to measure differences.  
You could, but not very successful.  
Because the difference between right and any green box  
image, the mean squared error really remain the same.  
It's two to five.  
But the visual quality, you see, it looks quite different.  
Therefore, we need a measure that can effectively  
reflect such a visual difference.  
That's the need, why we need a structural similarity.  
And for human vision system, we focus more  
on structural information.  
It's not a pixel-wise measurement.  
So remember, the mean squared error,  
you just calculate discrepancy pixel by pixel.  
Nothing to do with the neighboring.  
You only focus on pixel.  
Then you treat all the error components equally,  
add them together.  
Now we say human vision system pay attention  
to structural information.  
And also, human vision system highly  
adapted for contextual changes.  
Like a background, a small change,  
you immediately notice.  
So the human vision system is very, very advised.  
So this is part of human intelligence system.  
So classic way, mean squared error, do bottom up.  
So bottom pixel-wise.  
So pixel-wise, what's the error?  
Then you add them together.  
So error visibility, just a classic scheme.  
And the really better scheme is top-top.  
So you just try to extract the structural information,  
pay attention to global similarity.  
Once I read an article, one school of thinking,  
say human vision system not to start with small detail.  
It started with a global feature called a topological feature.  
If these big things are connected,  
how many whole separated components,  
those are very high-level property  
that wouldn't change if you stretch, scale, rotate.  
So the property doesn't change with these kind  
of scaling, rotation, operations.  
So those properties are higher-level property  
called topological properties.  
So high-level property, very important.  
How should we define structural information?  
So the example I show you, you can use this new measure  
to tell the differences.

Classic paper is image quality assessment from error visibility to structural similarity. This is the paper I'm going to explain. And the paper proposed a measure called structural similarity, or SSIM. So if you compute SSIM between reference image and any image, if you have a reference image, red box, then you have another image, still red image. They are identical, right? You input these two images into this box to compute SSIM. The value is 1. That means 100% is identical. If on one hand you have this red box, the other hand you put this blurry image, it returns a value only less than 70% is similar. You see this number between 0 and 1. The closer one, the more similar two images are. So this looks pretty good. You may wonder how this magic SSIM works. So structural similarity computed this way. So you have image X, image Y. Say it's two dimensional signal or three dimensional signals are called images. You have X and Y. First, you do luminance measurement. The luminance measurement, you got the measurement, and you subtract the measurement, make sure the mean is the same. Then you do the contrast measurement. So around the mean, the signal may be up or may be down. So you can compute the standard deviation. So once you compute the mean, you do division. So you normalize the signal with respect to the mean. So the result will be the image with 0 mean and the standard deviation Y. So you get a normalization. You do this normalization process for X, and also you do so for Y. So you compare luminance. You compare standard deviation here. Then after normalization, you compare structural similarity. Then the ultimate measure, SSIM, consists of three components. Luminance comparison and contrast comparison and the structural difference added together. That is called similarity measure. So this is the overall idea. Let me explain. So try to follow me about this idea. So we say structural similarity really has three components. Luminance, contrast, structure. ILCS, we're talking about a comparison between two images, X and Y. So for given image X, you have capital N pixel. You do average, you get mean. You remove the mean. So you do first level normalization. And once you know the mean, you can compute the standard deviation. So this is a standard way to calculate the standard deviation. And this factor is not 1 over N, rather 1 over N minus 1. So for some statistical reasons, this is standard deviation. Then you do normalization. So this is what I just mentioned. After normalization, the normalized image will have a 0 mean and the standard deviation 1. Because you normalize, you scale this right back to standard deviation. So this is some basic operation, very common thing.

And we will do first upon the images, X and Y.  
And to construct the similarity measurement,  
the similarity measurement is something like a distance.  
But the similarity more emphasizes  
how things are similar.  
So we just impose three postulates.  
We say the good similarity measurement  
should be symmetric.  
So say X is similar to Y in the same way Y is similar to X.  
It's just symmetric.  
Also, we add a bound.  
This is different from distance.  
So we just make sure that you have a perfect similarity.  
That's 100%.  
Otherwise, you have something less than 1.  
And this would put here greater than or equal to 0.  
And also, we request for uniqueness.  
So if similarity measure is equal to 1,  
then that means X is equal to Y.  
And whenever X is equal to Y, you  
must have similarity measurement equal to 1.  
So this is a necessary and sufficient condition.  
These things all look very reasonable.  
So we are constructing a similarity measurement  
to satisfy these three postulates.  
First, let's talk about luminance comparison.  
And the measurement is defined up front.  
It's just based on heuristics.  
The earlier researchers say, OK, let's just  
define luminance measurement as this.  
Just look at this.  
And initially, they didn't put the constant C1.  
They just say  $\mu X$  and  $\mu Y$  twice,  
divided by  $\mu X$  squared plus  $\mu Y$  squared.  
And you can see if  $\mu X$  and  $\mu Y$  are the same,  
then the value is 1.  
And if they are quite different, say one of them is very small.  
The other is very big.  
They are dramatically different.  
The value, the function L of XY will be very small.  
But the problem is that without the constant C,  
when both  $\mu X$  and  $\mu Y$  are small,  
you have a very big denominator.  
So this is not good.  
So they add a constant.  
And heuristically, they say C1 should  
be equal to squared factor.  
The factor is proportional to the dynamic range of the image.  
With a small constant K1, L is the dynamic range  
of the image or pixel values.  
Say the dynamic range for image of 8-bit grayscale image  
is 255.  
This is the first part.  
And then we say the similarity requirement.  
And we have this symmetric requirement,  
the luminance measurement to satisfy that.  
Any questions?  
Do you have any questions?  
Pay attention to lecture.  
And the boundedness and the uniqueness.  
And this can be shown to be true for this luminance comparison.  
Do not keep talking.  
Not good for your learning.  
And you can try to understand why this is so.



And I will not explain everything,  
just show you this simple idea.  
So we say the  $\mu X$  is A, and the  $\mu Y$  is X.  
Here A is  $\mu X$ . Here X is  $\mu Y$ .  
We keep changing the X. See how the value will change.  
Particularly, we say what would be the X value?  
That makes sure this measurement reaches maximum.  
So we just do first derivative.  
So this is the definition of luminance measurement.  
And we do first derivative with respect to X.  
So X is in the place of  $\mu Y$ . So just do first derivative.  
Then we set the first derivative to 0.  
And then we solve this, and then we have the conclusion  
that when X is equal to A, this value reaches maximum.  
So the maximum value is Y. So this is back up what I said.  
So when both means are equal, the value is maximum.  
So you can just follow this to understand better.  
So this is the first term.  
How you compare the brightness background  
level of two images.  
The two images are same or similar.  
Their means should be similar.  
If they are different, the mean can be quite different.  
So this is the first aspect.  
Second aspect, we should compare.  
Even they have the same mean, and one image  
may have larger standard deviation.  
The other one may have smaller deviation, standard deviation.  
So the two images are very similar.  
Then they must have a very similar standard deviation.  
So this is the second aspect defined in a very similar way.  
And then the constant is called C2 in a similar form.  
But just the coefficient becomes K2.  
First case is K1.  
And I explained already.  
So when  $\sigma X$  is equal to  $\sigma Y$ ,  
this function C of XY will reach the maximum value.  
The maximum value is Y.  
And this definition of contrast comparison  
and very consistent to human vision system.  
For human vision system, so very dark room.  
You have a very, very weak, small spot, small light.  
You immediately see.  
But if you have a very bright room,  
and still same weak light, you couldn't see.  
And just like in the daytime, the light night,  
you can see many stars.  
But you go out now, you cannot see any star.  
Because the background light is so strong,  
so you couldn't see.  
So human vision really detects the relative contrast.  
So you see the sensitivity.  
And it's really a function of the background brightness  
and the strength of the physical stimulus intensity.  
And about the change.  
So if you have a very light background,  
you need to change much more relative to very, very low  
background.  
You only need to change very light.  
And the hearing system, same thing.  
And if you have a very quiet room and a small sound,  
you can pick up.  
But when you are in, say, some party, a lot of noise,  
you need to shout it out to be heard.

So similar idea.  
So this can be shown simply.  
And it's not a systematic derivation.  
Just say you have some value, say  $\sigma X$ .  
And  $\sigma Y$  is basically similar to  $X$ ,  
but with a small change,  $\Delta X$ .  
How this small change will be reflected  
in this second measurement?  
So it's just do algebraic derivation.  
And you see the result is something like this.  
When you have a decent  $X$ , so  $X^2$  is very big.  
So this term is small.  
So it's just an egg note.  
And this term is big, your egg note.  
So what left over?  
So the second measurement really equal to  $\Delta X / X$ .  
So it's a matter about this result.  
It's not absolute  $\Delta X$ .  
So the contrast may be differ by 100.  
But if it is 100,  $\Delta X$  equal to 100,  
is on the background, say 10,000 would not  
be so much significant.  
But if it is  $\Delta X$ , 100 on a background,  $X$  equal to 1,  
that will be very significant.  
And to show you, to solve this idea further,  
look at this picture.  
So first column, so you have 10,000.  
The next one, your bottom one, you have 20,000.  
So the difference is 10.  
So if you just look at these two images,  
you immediately feel you have more dots here  
than the upper one.  
In the second column, you have 111 dots.  
And the bottom right, you have 120.  
The difference is still 10.  
But if you look at these two, so it's not  
as clear as in the case in the first column.  
So this idea, the change over background,  
the change is  $\Delta$  over background.  
This result determine how our vision system pick up signal.  
So we say this second measurement really  
works in a consistent way as our human vision system.  
So this is just the idea.  
Third component, you say, after you normalize the first image,  
with respect to  $\mu$  and the standard deviation,  
the first image and the second image,  
after you do normalization, now we  
compare structural similarity, how the two images,  
they look the same.  
And the researchers, the earlier researchers,  
they find the similarity as cross-correlation.  
Normalized by product of standard deviation.  
And this constant  $C$ , same consideration  
like we explained for  $C_1$ ,  $C_2$ .  
You have  $C_3$  here.  
So this is cross-correlation.  
And for normalized vector, it's shown here.  
You see here, the  $x$  component, it certainly  
normalize the  $x$  component and normalize the  $y$  component.  
And this is the same thing.  
I keep explaining to you, this inner product.  
So you have  $i$  equal to 1, 2, 3, 5 for  $x$ .  
 $i$  equal to 1, 2, 3, 5 for  $y$ .  
You do this matching.

So you've got all these partial product.  
Then you add it together.  
The result returns as the sigma xy.  
So how we say sigma xy is just the correlation coefficient.  
There's a correlation coefficient between x and y.  
So you read through this.  
So this is how they define the structural similarity.  
But the inner product, geometrically,  
I explained to you before, in high dimensional space,  
one vector really projected to the other vector.  
And the value will be maximum when  
the angle between the two vector.  
Each vector is an image now.  
So when the two image, they really have the angle  $\theta$ .  
They just become just one scaled version of the other.  
Then you have the maximum value.  
So we have a mathematical reason to define this structure  
similarity.  
So when we learn Fourier analysis,  
and I try to give you geometrical insight.  
And then from the examination, I know some of your data  
really well.  
So you got the insight.  
That's really great.  
Other students still got lost.  
It's always the case, some students  
do better than other students, just depending on topic.  
But this is the same thing.  
So the whole mathematical foundation  
and the really connected, the piece interconnected.  
Here, if you understand the inner product,  
the geometrical interpretation, you  
will understand why you do this definition.  
This is the same reason.  
How you compute Fourier coefficient,  
it's just a projection, one vector.  
Your image is a vector in a high dimensional space,  
projected onto here, onto another image  
you want to compare with.  
And in Fourier analysis, you project onto one basis vector.  
So basis vector altogether form also normal basis.  
Same idea.  
And mathematically, we learned this.  
It's just a copy of this slide called CSEquality.  
The C is inner product only reads the maximum  
when A and B are linearly related.  
One is scaled with the other.  
So just try to understand the idea.  
That's always important.  
Even we are engineers, we don't just do things just as a rule.  
We should know why it works and what's the idea behind it  
so you can become creative and do a better job.  
So after all these considerations,  
and we say, now we've got, as I am, three components.  
Luminance, contrast, similarity, each of them already defined.  
And the functions should rely on these three factors.  
And each factor, you can read to certain power,  
and that will determine relative importance of the aspect.  
But for easiness, let's just say alpha, beta, gamma,  
all equal to 1.  
Furthermore, we say C3 and C2, they more or less the same.  
C2 equal to half of C3.  
And this is just the empirical selection, no theory behind it.  
And after these convenient choices,

and we have this version of structural similarity. This is commonly used in the earlier version called the universal quality index without this C. But you better put C for the reason I mentioned. If all C coefficients are 0, when you have a very small mean of standard deviation, the measurement wouldn't be stable. So this is just an engineered image quality metric. It works very well. That's why different areas just use this measurement. Even now, we do image quality assessment with some state-of-the-art algorithm. We still use mean square and the structural similarity as quality assessment metrics. And just to give you another example, you see you have different images. Then you compare the images. And using the quality index, I just explained. So the first one is the right boxed one. It's just the ground truth. You have a different version. And with SSIM, if you read the number carefully, I think you would agree. This measurement really captures the essential aspect of the human vision system. So when you design a communication channel, or you produce images in comparison with truth, you use this number. You can just try to do optimization against this number so you have good results. And that will pretty much what radiologists or users want to see. So this is very important work. And because of the working principle, I explained very reasonably. So different areas try to adapt for color image quality assessment. And what I explained to you, just the grayscale, no color component. And you can consider how to extend that measurement for color image evaluation, or video quality assessment, even audio quality signal-wise. Or you do multi-scale analysis. Later on, we will learn MRI imaging. MRI images can be represented as complex-valued images. Then you would have the question, how you evaluate complex-valued images? How they are similar? So all these represent follow-up work. I explained the idea I explained to you. So this is not required. I just mentioned this is a very good image quality measurement, better than mean squared error. So somewhere here, let me stop for a second talking about your first exam. You see the SSIM. You first do normalization with respect to mean  $\mu$ . Then you do normalization with respect to standard deviation  $\sigma$ . So what do you want to see? You want to see you have a certain expectation. Then you want to see the variation around this. And the variation around the mean, you can measure as a variance or as a range.

When we do scoring, grading your exam or homework question, usually we say your best result is 100. Normally you wouldn't get 100, but in the best case, you get 100. The per is 0. So the mean, roughly, should be just 50. This is 50, this is 100. So I believe a good exam should be designed so the class will have mean 50, not mean 90. That's not good, because we want to tell differences. And the examination is to probe into your learning results. This is ideal mean, in my opinion. So what do you mean in the first examination? So again, the mean is just for evaluation. So at the end of the day, you will have score distribution. And you gather this, it just means you are average. That does not mean you fail. If you think all the way, 60% is a passing line. It's not the case for exam 1, 2, 3. But at the end of the day, we do curving. So don't worry about that. So TA sent me the results. I think this is kind of what I expected. So this is below 40 to 50, below, this is above. So somehow the mean is around that, a little bit more. So this is what we did several days ago. And a year ago, so this is, say, where is 50? Here, here is 50. 10 is 50, so you got this score. But at the end of the day, this is a little better. The questions are different. But at the end of the day, after curving, so this is a great distribution we have, say, two years ago. And then each year, last year. And also this year, for your class, the grade distribution. So the final day, I will use the curving. So we'll look normal to the common practice. But do not feel bad if you got 50. 50 means you are average. So this is just some explanation. I think the score distribution like this, the majority, really you'll pass. The worst case, two students really fail. And this is less than 60 mean just a simple, very simple thing. This student escaped class, didn't do well. Unless you surprise me, you wouldn't fail. So at the end of the day, you should expect a similar distribution. So this is what I would like to explain to you. How you interpret your score. And the most important thing, you try to follow the lecture, and you will do well. The second part is system-specific measurement. I think now I give you about 10 minutes, rest of the time. Then we come back to finish the last two part. Let's continue. So talking about score, I think you got 50. Above 50, that's reasonable, I think. But if you're below 50, please work harder. And you got above 90, that's really good. I need to talk to you sometime. Anyway, second part, system-specific measures. First, talk about signal to noise result.

So in literature, you oftentimes see for engineer, SNR is a very familiar terminology. So you have a background noise to take picture. You always have noise. So noise is inherent in physical measurement process. After all, so everything can be explained by quantum mechanics, nothing for sure. So statistical nature is very fundamental. And on top of noise, you have signal. The signal noise result is a heuristic measure. It just do reasoning between signal power and noise power. The power is proportional to squared amplitude. So think about Fourier analysis. Anything you decompose into sinusoidal component. For single sinusoidal component, like AC current, power is amplitude of AC current and some proportional factors. So this result shows how strong the signal is relative to noise. If the result is 1, so the signal varies same range as the noise. So it's barely visible. If you have a signal noise result over 5 or over 10, you can easily detect the signal. So this is a concept, a related concept I put in the book draft, didn't include here. It's called contrast to noise result. Contrast, you have, say, signal 1, signal 2. Signal 2, maybe you're background. So signal 1 minus signal 2. So absolute difference. Then divided by amplitude of noise, noise background. There's a contrast to noise result. Also very common. And the image resolution. So resolution, I said, can be decomposed into several ways, several aspects. So the first type of spatial resolution are called high contrast resolution. Talking about high contrast details, say, two bright dots. So one put here, one put here. So if they are separate, and each dot is something like a delta function. But after imaging precise, the delta function, a small bright dot, becomes a blurry disk. And the small dot is a mathematical abstraction delta function. And you cannot get a delta function with any imaging system. So what you get is a blurry spot. That is a system's response to delta function, called a point spread function. A bright point becomes a Gaussian-like blurry, like here. So this is called a point spread function. It's an important concept, the point spread function. So you may write down. So PSF. So you have a single dot here. You'll have another one. You can clearly see there are two dots. Although blurry, but you can still say, OK, two dots, like two tumor. You just report two tumor. But see, when the two blurry spots

getting closer, closer, closer, you  
imagine eventually they will be fused into one.  
So the critical moment is the separation  
is equal to full width at half maximum of point spread  
function.  
When the point spread function is merged with separation,  
full width at half maximum.  
So see this intersecting point at half maximum.  
But you have this combination.  
So two half maximum value added together.  
So you've got this value equal to maximum.  
The maximum is really maximum of any Gaussian point spread  
function.  
So their boundary is as high as the peak of either spot.  
So the two spot appear one.  
If we get closer, you still cannot see.  
So that's very reasonable to think image resolution  
is a minimum separation.  
So less than that, the two dots become one.  
You cannot see.  
So this is one way to define image resolution,  
or high contrast resolution.  
Oftentimes, simply say resolution  
mean this detail resolving capability.  
Another way to view spatial resolution  
is to calculate so-called modulation transfer function.  
Say you have dots, and then you just  
perform Fourier transformation.  
So a dot is a point, is a delta function.  
You have many Fourier components.  
Some are at a lower frequency, some at a higher frequency.  
So you think the system responds to sinusoidal input.  
So for low frequency input, it just passes through.  
And the higher frequency, the gain becomes less than one.  
Very high frequency, that means the sub-eyes  
cannot pass through easily.  
So the eyes get blurred.  
So you've got less and less response.  
So you perform Fourier transformation.  
And you take a modular operation,  
meaning you just forget the phase,  
calculate the amplitude of the sinusoidal function.  
We do normalized computation.  
See how easily sinusoidal components  
go through the imaging system.  
The higher frequency, you just have a stronger attenuating  
factor.  
So you go from 100% all the way down.  
And for high frequency, you think  
this sinusoidal component, or you  
think the line pair phantom, you have one bar, max bar.  
So just you have one bar, you think high is one.  
The middle is zero.  
So one, zero, one, zero.  
One, this bar is just a approximation  
of the sinusoidal components.  
When frequency is very high, the period  
is very short, the resolution is measured  
in terms of line pair per millimeter.  
And in sinusoidal terminology, it's cycles per millimeter.  
So when you have high frequency and many cycles,  
or many line pairs per millimeter,  
the system starts blurring them out.  
So you fail to resolve line pair.

That's a limited system resolution.  
Usually, you let the modulation transfer function decay until to a point below the noise flow.  
So noise background is a high frequency oscillation.  
So when the high frequency gain, the high frequency component after passing the system is below noise flow, to draw noise flow, and then the intersection point here corresponds to highest frequency, or highest number of line pair per millimeter above the noise flow, or close to noise flow.  
So any higher frequency goes down the noise flow, the system couldn't resolve.  
This is another way to measure system resolution, called modulation transfer function.  
There are many ways, but these two measures are very common.  
And the resolution has many types.  
So spatial resolution, also called high contrast resolution.  
The next one is called low contrast resolution, simply called contrast resolution.  
This is not talking about resolving bright high contrast structure in a neighborhood.  
Rather, it talks about the capability of the imaging system to tell a part, subtle seeding different.  
And the structure may not be small, may be big, but can you tell that you have a structure in the background?  
The seeding can be very small.  
So this is talking about contrast.  
Like if you see very carefully here, you see some small days, but in noisy background, you cannot tell them apart.  
So usually, we measure this capability.  
The small contrast detail can be picked up.  
The structure is not small, it's big.  
And this is clinically important, like a tumor detection.  
And the tumor and the biological soft tissue, they all made up a light chemical element.  
And the CT number look very similar.  
But some system allows you to see the structure better.  
You can detect the barrier.  
There is this here.  
In clinical setting, you may be able to see.  
OK, here is a tumor.  
And the other system, very noisy.  
You just see noisy background.  
You couldn't detect the tumor.  
So the capability in this regard is called contrast resolution.  
I really need to put two more slides to briefly mention temporal resolution and spectral resolution.  
But let me just verbally explain.  
These concepts are easier to understand.  
Temporal resolution, they say how quickly you can take a picture.  
You take a snapshot, you get a picture crystal clear.  
It's a good picture, high-speed camera.  
And on the other hand, you can have a slow camera, or the object moves too fast.  
You get a blurry impression.  
But you take a picture, the bullet traveling, you cannot capture it.  
Just see a line, if you use camera speed very high, but not high enough, you will see motion blurring.



Like you take a picture, a moving car, use iPhone,  
you take a moving car, you see blurring  
along the motion direction.  
The most blurring you'll see.  
So that's about motion blurring.  
If the system can take a crystal clear, instantaneous picture,  
we say the system has a very good temporal resolution,  
or time resolution.  
The spectral resolution is a third, third,  
is a false aspect.  
Say high contrast, low contrast, temporal, and spectral.  
Spectral resolution is a false aspect of resolution.  
And this is easy.  
You see, some of us, maybe hundreds of us, maybe one  
or two, are color blinded.  
And those not color blinded, you see colorful picture.  
For color blinded, you see no color.  
The spectral resolution is not good.  
For imaging system, like Doppler imaging,  
you use a high frequency, low frequency.  
X-ray imaging, you use a high energy X-ray, hard X-ray,  
or low energy X-ray.  
The frequency really related to color,  
because the human vision, you see colorful images.  
So red, blue, green, all related to frequency of EM wave.  
So frequency is color.  
So how accurately you can resolve frequency components.  
You tell, say, certain frequency range from the other range.  
The narrow frequency range you can  
resolve, we say you have a better spectral resolution.  
Fourier analysis and the resolve frequency components.  
So the Fourier analysis is related to spectral analysis.  
And the spectral band, say, the energy range.  
And right now, the X-ray energy resolution is about 10 keV.  
And you can convert that to frequency range.  
We will explain more in the X-ray lectures.  
Anyway, we talk about noise background.  
Signal over noise, called the signal to noise result.  
And the contrast difference in signals over noise background,  
contrast of the noise result.  
We talk about resolution, four kinds of resolution.  
Spatial or high contrast resolution.  
Low contrast resolution, or simply contrast resolution.  
Temporal or time resolution, spectral resolution.  
So last part for system specific measurement, image quality  
measurement, is artifacts.  
Artifacts are ghosts.  
You see something like a holographic picture here.  
You raise nothing there.  
That's a ghost.  
So in tomographic image, the imaging process,  
later you will learn.  
It's very complicated.  
Like we take CT images.  
And oftentimes, we don't know how hard it beats,  
particularly when you have cardiovascular disease.  
You don't know exactly how hard it beats.  
When you take data and you make an image,  
you assume how hard it is stationary.  
So this model mismatch will cause a problem.  
When you reconstruct images, like motion blurring,  
and like different things, and you see some structure.  
And then you can use ultrasound imaging.  
You get a multiple echo back.

And the echo bouncing back and forth.  
And after reaching the transducer,  
you see multiple dots, but actually only one dot.  
But you have some other structure bouncing the signal  
back and forth.  
So those things are not real artifacts.  
And I'll give you one example.  
For the metal artifacts, in human body,  
when you break your bone, and we put metal implant  
inside the hip, like you put two metal implants.  
When you shoot x-ray from top to down or the other way,  
the metal part is very dense.  
You block x-ray.  
But the reconstruction method, assume  
you have data available along this direction,  
and also along the path going through the metal region  
where the data are missing or seriously compromised.  
But the algorithm doesn't know.  
Just keep doing the regular computation.  
The result is that you look this straight.  
You do not have a band inside the body.  
So this structure is not real, called just metal artifacts.  
So this is a simulation result. This is ground truth.  
You really should have a structure look like this.  
But due to the metal part and the metal artifacts,  
you couldn't see things between the implants.  
And you can use some algorithm to do metal artifacts  
reduction to get an image a little better.  
And then we further use neural network machine  
learning to improve the metal artifacts reduction result.  
We got an even better image.  
This is case one.  
Case two.  
So just to let you know, what's the artifacts?  
The artifacts depend on imaging modality,  
depends on algorithm and the application specific,  
many kinds.  
But this is just the concept you know for the moment.  
So second part, we just finished about imaging system  
specific measures, usually summarized in specific cases.  
When you talk about the imaging system,  
you go to some conference exhibition.  
And you see manufacturer.  
You may say, could I have your system specification?  
You will hand you a brochure.  
So you flip over, you say image resolution, noise level,  
reducing dose, imaging speed.  
Let's talk about temporal resolution.  
Reducing dose and all the things put into system  
specifications or specs.  
So this is about system.  
Now the last part, and the most important, all the imaging  
systems, we really do not, in a way,  
we do not care how good the resolution, speed.  
We do not care these things directly.  
All these will be used to deliver correct  
diagnostic decision.  
If we can do diagnostic job successfully,  
the resolution, good or bad, doesn't matter.  
Really just as long as we can use it to serve our purpose.  
So now we cover about clinical task-based image quality  
assessment or imaging workflow assessment.  
So this is a typical radiology department.  
So doctors, with years of training and experience,

look at the image and just make sure  
if this smoker has a lung tumor, if the cardiovascular structure  
look fine.

So all these are in clinical terms.

So we need task-specific measures.

So let me give you a simple example.

You see the difference between task-specific measures  
and the system-specific measures.

And this is an example, the tower.

So you do segmentation.

So basically, this result, you have two classes.

Either you have this edge or background, not edge.

Then you do detection.

It's edge, you think, like a tumor, some clinical target.

You want to detect it.

Then you only have four possibilities.

So you have a tumor.

Then you report, OK, here is a tumor.

They call the true positive.

You have a positive reporting.

You have detected the tumor.

You have a normal patient, and no tumor at all.

Then you do CTE or MRI or nuclear.

Then you say, OK, you're fine.

No tumor.

It's true negative.

True positive or true negative, it's all truth,  
correctly reported.

So the imaging system or the doctor,  
together with the system, doing fine job.

And the two kinds of errors, we do not want to have,  
but we are not God.

So we sometimes we make mistake.

So two kinds of mistake.

So you have a tumor, but you say, OK, you are fine.

No tumor.

You don't have tumor like a woman do mammography.

They say, you have something weird, looks like tumor.

Make the lady very nervous.

There's a false positive.

So false negative could be even worse.

And you have tumor.

You lose the opportunity to do early treatment.

And a false positive is not pleasant either.

It can be very burdensome.

You worry.

You think a lot of things unnecessarily worries you.

You feel, I may be dying today, something like that.

But maybe the big tumor is just the size,  
just the kind of liquid, really benign.

You take out, you are fine.

But you feel, OK, I have big mass.

I'm going to die.

You worry about your kids, and so on.

So all these things, when we think  
about the clinical situation, we want  
to fight for true positive, true negative.

We want to reduce false negative, false positive.

So two terminology.

Oftentimes, if you talk with a doctor for treatment,  
they say there are some new drugs or imaging modalities  
that detect something.

They talk about the sensitivity is not very good.

The specificity is not very good.

And all the sensitivity is really good,  
but the specificity is not good.  
So the terminology has a special meaning.  
And they're very confusing for me for years.  
So I try to remember these things.  
OK, this is a false situation.  
TP, true positive, FN, true negative, FP, false negative,  
and the false positive, FP.  
So these are easy to remember.  
And now we are ready to define what's the sensitivity.  
Sensitivity is not just the ordinary terminology.  
I am not so sensitive to code.  
And please be more specific.  
In medical domain, for task-based evaluation,  
we have a formal definition.  
So sensitivity means true positive,  
number of true positive report divided  
by sum of true positive and false negative.  
That means you really have disease,  
but wrongly reported as a fine.  
So this is a likelihood of a positive case,  
or percentage of condition of disease we find.  
In other words, we say the sensitivity  
is about how sure we say yes.  
We say you have a problem.  
You have a problem.  
Maybe 100 people, you have a problem.  
And then you do say, not all of them, you reported.  
And some people do not have a problem.  
100 people are healthy.  
Then you say, OK, there's a false negative.  
100 people with a problem.  
Let me back up, say 100 people.  
And 50, 100 people have a problem.  
And these people go through imaging study  
or radiological reading.  
50 of them, they say, OK, you got a problem.  
That's called true positive.  
The other half of 50 patients, and luckily, they  
were told it's a false negative.  
So you say you have no problem, but it's not the case.  
So you have 50, 50 together.  
Here is 50, so sensitivity is only 50%.  
So this sensitivity is how sure we say yes.  
So just try to remember the second letter is E.  
And the specificity, also you try  
to see the second letter, P, is how sure we say no.  
So definition is true negative.  
Whenever you have this imaging study,  
you can calculate true negative and false positive.  
There's true negative and false positive  
added together as denominator.  
And the numerator is true negative.  
So this is just the false negative.  
Many people come out and say, I'm negative.  
So all these people come together, a pool.  
And among them, all these claim to be negative.  
How many of them are really negative?  
That's called specificity.  
How sure, when you say negative, you say no, no problem.  
How sure it is.  
So this is a definition about the sensitivity  
and the specificity.  
Seems a little tricky.

Several concepts, even for me, working in medical area for decades, and still need a review. And it's just something very confusing, I think. And it makes things even more confusing. Let me introduce two more concepts called positive predictive value or negative predictive value. PPV and NPV.

So the numerator here for PPV is true positive. Denominator is true positive plus false positive. So all these things, true positive, false positive, means those you say you got problem, but those people really do not have a problem. So you define this way. And the negative, similarly, the denominator is the sum of number of true negative plus false negative. So all negative people added together.

Let's just look at an example.

You try to detect a tumor.

Here is not tumor.

It's called tuberculosis.

So you do actually detecting.

So you have a number of cases.

Total is nearly 2,000.

So the positive patient, and you reported positive, and you reported negative.

So among 30 patients with the disease, 22 of 30 were reported positive, and 8 of 30 were reported no problem.

And the second column, so among the patient, majority of patients do not have problem.

And 21 of them were incorrectly reported a problem.

You may see some doubt, but it's a normal structure.

You think this is a problem.

You reported it so.

And the majority of normal patients were reported no problem.

So this is just a table.

So you have the four cases shown here.

You do a marginal sum here.

Then you have a total.

This is total number of patients.

According to what I explained to you minutes ago, you can compute the sensitivity.

20 is a positive.

You reported as a positive.

Among all patients who got a problem, so you do the result. You got the sensitivity.

But for positive predictive value, the numerator is still 22.

That's the number of cases reported as positive.

But the denominator is all the positive report, including those patients with problem, without problem.

So the denominator is 73.

So you put 73.

Here, denominator is 73.

Here, denominator is 30.

So that's the difference between sensitivity and the positive predictive value, PPV.

So you look at this example.

Really nothing challenging.

We are not familiar with this definition.

So it makes things a little confusing.

We use the terminology, sensitivity, specificity, and the predictive values.

And you're kind of heuristic, but we

have a specific definition.  
So try to remember this definition.  
And the diagnostic accuracy, 22 is a yes.  
And 1,739 is a no, is a no.  
So yes is yes, and no is no is what we're supposed to do.  
So we do a good job.  
So this is correct report we produced.  
This correct total number of correct reports  
we made and divided by total number of patients  
is our diagnostic accuracy.  
And nearly about 95% is nearly 97%.  
The rest, the 3%, we made a mistake.  
Two kinds of mistake.  
One is false alarm.  
You are fine, but I say, OK, you have problem.  
And the other is you miss the signal.  
So you got a problem.  
It just gave you false assurance.  
Say you look fine.  
Don't worry.  
So the rest, the 3%, is a problem.  
Positive, you say no.  
And the negative, you say yes.  
And neither is good.  
So looking at this example and this concept,  
sensitivity, specificity, and predictive values,  
diagnostic accuracy, prevalence is  
how common the disease is in the population.  
In this population, nearly 2,000 people, only 30 of them  
got problem.  
That's a percentage of disease.  
So you use this terminology, prevalence, PR.  
So it's clear.  
Looking at this, it's clear.  
OK?  
So now we are familiar with these task-based image  
or imaging evaluation, reader study concept.  
And we can go a step further.  
Let me introduce a very nice figure called receiver  
operating characteristic.  
So this curve is very informative.  
It's smart design.  
And as we go through the next few slides,  
you will get a better idea.  
So basically, specificity and sensitivity  
are two initial concepts I explained to you.  
Sensitivity is the measure how able you do reporting  
so that yes is reported as a yes.  
And specificity is how capable you can say no for no.  
So this is yes, yes, no, and no.  
These are two aspects.  
So we just make a curve.  
These are related.  
And then you can just make sure.  
Yes, you always say yes.  
You can also make sure no, you say no.  
For this whole group, I can just claim all you guys have cancer.  
They just say everything I say.  
So then by saying so, I made sure in any real case,  
I didn't miss anything.  
No signal I will miss, but a lot of false alarm.  
On the other hand, I can say all you guys  
are in perfect health condition.  
So I never give you a false alarm,

because I say, oh, you're fine.  
But I do miss some important cases.  
A case is that some student here may got a problem,  
but I just say you have no problem.  
So these two things are connected together.  
You need a balance for different.  
You can select a specificity.  
Then you'll find the corresponding sensitivity.  
So you change the specificity, and then you  
have this curve traced.  
Overall performance is not by a specific selection  
of specificity or sensitivity, really by this curve.  
So if I just challenge you at a different level of sensitivity  
or at a different level of specificity,  
what is the corresponding, the other measure?  
So the other measure and the specified level  
of the first measure altogether define the curve  
when you keep changing sensitivity or specificity.  
And the area under the curve is a measure  
of the diagnostic performance.  
And this may not be crystal clear to you,  
but it will become clearer and clearer to you.  
So this is still the same thing.  
So  $1 - \text{specificity}$ .  
If you check the original definition of specificity,  
so  $1 - \text{specificity}$  is a false positive fraction.  
And this is a true positive fraction.  
So the true positive and the false positive together,  
what's the fraction of a true positive?  
That is a true positive fraction is a sensitivity.  
This is a  $1 - \text{specificity}$ , false positive fraction.  
So this is another view of the same thing.  
Just you need to see it from a different point of view.  
Let me give you an example to see how you form this curve,  
how this curve could change.  
And you prefer the curve is stretched  
towards this top left corner.  
You want to stretch this curve towards this.  
So pay attention to next few slides.  
I think after that, you will be clearer.  
You have statistical distribution, like the tumor.  
You have the non-diseased and diseased case.  
And a certain feature, you measure.  
Ideally, you want the feature to be clearly different.  
Say, non-diseased, so like the certain value,  
like the diameter of, say, the vessel, so on,  
will be in this range.  
The disease that you got deletion  
got a little bit bigger, or the size of tumor  
a little bit bigger, in this range.  
Then you put the threshold between,  
and you got a perfect situation.  
I can say I'm in perfect situation here.  
I can say whenever you're in green area, I say, yes,  
you have problem.  
In red area, I say, no, you don't have problem.  
So I have a perfect capability to do what I said,  
yes is yes, no is no.  
This is an ideal case.  
Therefore, we can extract the feature so clearly.  
This idealized situation is normally not the case.  
And the normal case is always overlapping situation.  
The feature is not totally separable.  
So you have a non-diseased, diseased,

say, just do some blood testing, one number.  
So diseased in this range with a lot of number around the peak.  
And the non-diseased in this region,  
diseased in the green region.  
So they got overlap.  
So if you have a number here, it could be from a patient  
with disease, or could be from a patient without disease.  
So that makes things harder.  
So in this case, suppose just this overlapping situation,  
I just put threshold here.  
Threshold here, so you have a disease,  
then I have a half chance to report you have a disease.  
It's a half chance I report.  
I say, yes, and yes, with sensitivity, 50%.  
So sensitivity, 50%, as I hear.  
What's the capability to say no is no.  
Or this is a false positive fraction,  
1 minus specificity.  
You see this a little bit.  
Why I set the threshold here?  
A patient come, and this patient come,  
but I got a number in this range.  
I will say, OK, you got problem.  
This area, right area, is the percentage or the probability  
for false positive.  
False positive rate or false positive fraction.  
We got this small part.  
This small part, say this  $\theta$  is 100%.  
This small part is somewhere here.  
So if I put a threshold this way, I got a point here.  
Let's start here.  
This is a less-graceful decision-making rule.  
So half a patient who have problem will be reported as old.  
You'll have 50% miss those diseased patients.  
OK, let's do moderately graceful diagnosis.  
So I move the threshold a little bit leftward.  
More than half a patient will be detected this way,  
so the green area.  
So the sensitivity a little higher.  
You got here.  
Then at the cost of this higher sensitivity,  
the cost of the higher sensitivity  
is that more patient, non-diseased patient,  
no more subject, will be reported as diseased.  
So this right area will increase.  
So false positive fraction is increased a little bit.  
Say you move from this black cross to this yellow cross.  
You move to this point.  
Let's do more graceful decision-making strategy.  
So majority of patients who are diseased  
will be reported as diseased.  
So I wouldn't miss a lot of diseased patients.  
And if you have problem, very likely you will be detected.  
But the cost for this higher likelihood  
is that about half a patient, half a subject,  
who have no problem at all will be reported got a problem.  
See, the false positive fraction or 1 minus specificity  
will be 50%.  
You will see 50% is right area, half of the Gaussian curve.  
So you got 1 minus specificity, 50%.  
But you got quite a high sensitivity.  
You got here.  
So things like this, you imagine.  
You just slide the bar continuously.



You will trace along this curve.  
And if you make the threshold far towards the left,  
that's the case I mentioned.  
I just claim, oh, you guys have problem.  
It's very easy for me.  
So I got a perfect number.  
At this point, I got 100% false positive fraction.  
And I got 100% sensitivity.  
So you just got things like this.  
So this is a whole ROC curve.  
So the best ROC curve is the one will go this way  
and go that way.  
So if you just draw the diagonal,  
that's just a random guess.  
Anyone come in, I flip a coin.  
I see the head, I say, you got a problem.  
And I see the tail, I say, you are fine.  
Then I will have ROC curve just move from this bottom left  
corner to top right.  
So diagnostic performance are not the same for all doctors.  
So if you can separate well, you'll  
get a better ROC curve, something like this.  
If you just couldn't see them at all,  
you just see the features.  
Disease, the not disease, they're all left together.  
You're kind of doing random guessing,  
going along the diagonal.  
So reader scale or radiologist expertise  
is reflected by area under the curve.  
So you move towards this anti-diagnostic direction.  
You are making yourself a better and a better doctor.  
And see this distribution, why different doctors  
have different performance, the same thing.  
But it depends on the training and the talent  
of an individual doctor.  
They see same information differently.  
A good doctor can see how better this discriminating power.  
They see features this way.  
Some naive resident will see this way.  
For me, I'm not trained.  
I do random guess, 50-50, something like here.  
Any diagnostic decision along this line  
has no predictive value because you just make a random guess.  
Actual test will be something like this.  
Ideal test go along the diagonal.  
Cannot be done either because in practice,  
you always have a chance to make a mistake.  
In daily life, you see doctors, they all wear white coat,  
look very professional, just like all students sitting here.  
But I give you one example.  
Some students do really well.  
Some students do poorly.  
Instructor-wise, some students, some instructor  
know the subject.  
Some other instructor may not know that well.  
Here is a real example.  
So just see the diagnostic performance survey  
among over 100 US radiologists.  
So you see they spread everywhere.  
So I would say Dr. Peer, in this region, is really good.  
But in this region, it's not so good.  
So finding a good doctor makes a tremendous difference.  
Just like a student find a good professor  
or professor find a good student, hire a good student

in the lab, the performance can be dramatically different.  
See, another example, the top radiologists,  
their ROC curve along this way.  
So they seldom make a case, always detect a problem.  
And other mediocre radiologists, like a radiologist resident,  
they are still training some not so good radiologists.  
They do a little better than random guys, but not by much.  
So this is nice to know.  
OK.  
OK, and we have a few more minutes  
to talk about an observer, numerical observer.  
So radiologists try to read the image and make a decision.  
We try to automate the process.  
Then we model the imaging process.  
And the idea is not that hard.  
Imaging system, so it's an image.  
From a real image, then there's a linear system operator.  
The matrix operator plus noise component.  
So this is an imaging model for majority medical imaging  
systems.  
This is a linear system.  
The fundamental image, you let it  
go through a linear system.  
So linear system, you have matrix and multiplication.  
So the image vector times the matrix.  
So you have all linear operations.  
Linear imaging system.  
Then you have two hypotheses.  
One is that you only have a normal structure background  
without any signal or any problem.  
The other is on top of the background,  
you have a signal or diagnostic feature.  
All you need to do is to do testing.  
So you have the image  $G$ . So you just  
see the probability for this hypothesis is  $\theta$ , normal.  
Or  $H_1$ , that's abnormal.  
So given the condition, if it's normal,  
what's the chance you see this particular image  $G$ ?  
And if it is abnormal, what's the chance?  
You just do this result.  
So this result is really critical.  
You just decide based on which probability is high.  
So this is an idealized observer.  
So idealized observer really requires  
all the statistical knowledge that normally you do not have.  
So if you just make simplification,  
you just use a linear observer.  
So you linearly combine the image into a number of features.  
Then you try to maximize the signal noise result.  
That is called hotline observer.  
And you get some features, not all the features,  
called channel hotline observer.  
This part, as I said here, it's just for your knowledge.  
So I wouldn't explain too much.  
If you're interested, you can just look at the slides  
carefully, get a good understanding.  
So linear observer is classic.  
And nowadays, radiomics image analysis  
uses machine learning using nonlinear operators.  
Nonlinear operators build with neural network,  
like this XOR operation.  
It's a nonlinear network.  
And now it's an emerging trend.  
You can use neural network, build a very complicated

nonlinear observer, get a very good diagnostic performance.  
And if you're interested, you read.  
But this part, observer part, is not required.  
So it just goes through quickly.  
And your homework question shown here.  
And I will upload the slides this afternoon.  
So now it's 1.50.  
We are done.